Coding for Leveraging Network Gains in 5G

Jörg Kliewer

The Elisha Yegal Bar-Ness Center For Wireless Communications And Signal Processing Research
The Zettabyte Area

[Cisco Systems Inc.: The zettabyte area. White paper, 2015]
How Can Cellular Systems Keep Up?

[Nokia Networks: Looking ahead to 5G. White paper, April 2014]
Recent advantages in communication and information theory constitute promising approaches to leverage **network gains**

- network capacity
- cooperative and opportunistic communication
- improved multiple access techniques
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**In this talk:** How to leverage network gains for both error correction and compression with modern graph based codes?
Improvements for error correcting codes have been limited mostly to the point-to-point case

- Low-density parity check codes
- Spatially coupled codes
- Polar codes
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Gains in transmission power efficiency to be expected from

- Coding for spectrally efficiency communication
- Multi-terminal coding and decoding (i.e., for relaying, cooperation, broadcast)
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Gains in transmission power efficiency to be expected from:

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Solution: Nested Codes
Nested Codes
Nested Codes

- Partitioning into subcodes $C_{\ell}$, $\ell = 1, 2, \ldots, M$
- Can be seen as structured linear binning schemes
- Finite-field version of physical layer superposition codes
Multiple access channels

- data from each source node is encoded by a subcode $C_{\ell}$
Applications for Multiterminal Communication

- Multiple access channels
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- Broadcast channels
  - Each destination node decodes a subset of subcodes $C_\ell$
  - type-1 nested polar codes achieve best known inner bound [Marton 1979], but with insufficient finite block length scaling
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- Interference channels
  - e.g., scheme from [Han & Kobayashi 1981], 2-user channel: message split up in public and private part (codes $C_1$ and $C_2$)
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- Many more: relay channels, cooperative diversity, wiretap channels, …
Consider $k$ by $n$ generator matrix $G$ of linear code $C$

- $M$ information words $i_k$

- **Type-1** codes: Partitioning of $G$ into subcodes $C_\ell$ with generator $G_\ell$ and rate $R_\ell = k_\ell/n$
Consider \( k \) by \( n \) generator matrix \( \mathbf{G} \) of linear code \( \mathcal{C} \)

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**Type-1 codes:** Partitioning of \( \mathbf{G} \) into subcodes \( \mathcal{C}_\ell \) with generator \( \mathbf{G}_\ell \) and rate \( R_\ell = k_\ell / n \)

\[
\mathbf{c} = [\mathbf{i}_1, \mathbf{i}_2, \ldots, \mathbf{i}_M] \quad \mathbf{G} = [\mathbf{i}_1, \mathbf{i}_2, \ldots, \mathbf{i}_M] \\
\begin{bmatrix}
\mathbf{G}_1 \\
\mathbf{G}_2 \\
\vdots \\
\mathbf{G}_M
\end{bmatrix}
\]
Consider \( k \) by \( n \) generator matrix \( \mathbf{G} \) of linear code \( \mathcal{C} \)

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**Type-1** codes: Partitioning of \( \mathbf{G} \) into subcodes \( \mathcal{C}_\ell \) with generator \( \mathbf{G}_\ell \) and rate \( R_\ell = k_\ell / n \)

\[
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\]

\[
\begin{bmatrix}
\mathbf{G}_1 \\
\mathbf{G}_2 \\
\vdots \\
\mathbf{G}_M
\end{bmatrix}
\]

**Type-2** codes: Partitioning of parity check matrix \( \mathbf{H} \) into subcodes \( \mathcal{C}_\ell \) with parity check matrix \( \mathbf{H}_\ell \)

\[
\begin{bmatrix}
\mathbf{H}_1 \\
\mathbf{H}_2 \\
\vdots \\
\mathbf{H}_M
\end{bmatrix} = \mathbf{H}
\]
LDPC Block Codes

Uncoded BPSK
Bit error probability
Shannon limit
Uncoded BPSK
Waterfall
Irregular LDPC-BC
Error floor
Regulated LDPC-BC

$E_b/N_0 \text{ (dB)}$
Tanner graph (3,6) regular LDPC code:

\[
\mathbf{H} = \begin{bmatrix}
1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

Graph sparsely connected
Coupling construction via unwrapping:

Convolutional code structure

[Costello, Dolecek, Fuja, Kliwer, Mitchell, Smarandache, 2014]
### Spatially Coupled LDPC Codes

**Coupling construction via unwrapping:**

\[
H = \begin{bmatrix}
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

\[
H_{cc} = \begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

Resulting Tanner graph:

Convolutional code structure

[Costello, Dolecek, Fuja, Kliwer, Mitchell, Smarandache, 2014]
Spatially Coupled LDPC Codes

Coupling construction via unwrapping:

\[ H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \]

\[ H_{cc} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \]

Resulting Tanner graph:

Terminated Tanner graph:

Convolutional code structure

[Costello, Dolecek, Fuja, Kliwer, Mitchell, Smarandache, 2014]
Spatially Coupled LDPC Codes: Performance

![Graph showing Bit error rate vs. $E_b/N_0$ for (3,6)-regular LDPC-BC and LDPC-CC with $n = 4098$ and $n_s = 4098$.]
How can we build good nested codes with spatially coupled LDPC codes?
Protograph representation of a type-1 nested spatially coupled LDPC code ensemble for $M=2$
Results

BER after decoding

$E_b/N_0$

$C$, random LDPC-BC
$C$, algebraic LDPC-BC
$C$, random SC-LDPC
$C_1$, random LDPC-BC
$C_1$, algebraic LDPC-BC
$C_1$, random SC-LDPC

random LDPC-BC
$n = 3248$

random SC-LDPC
$\nu_s = 2454$

algebraic LDPC-BC
$n = 3248$
**Example:** Distributed fronthaul compression for cloud radio access networks (CRANs) in 5G

[Image of a diagram showing cloud, CU, and multiple RUs (RU 1-8) connected to each other and to the cloud.]

[Park, Simeone, Sahin, Shamai, 2014]
Little attention has been paid so far on how data compression can reduce the network traffic.

Practical network based compression approaches virtually unknown.
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Practical network based compression approaches virtually unknown.

In the following: Lossy compression based on spatially coupled low-density generator matrix (LDGM) codes

- Low encoding and decoding complexity (linear in time)
- Performance very close to the rate-distortion limit
Idea: Treat source sequence as noisy codeword from some fictitious channel code (here a spatially coupled LDGM code)
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Source encoding via modified belief propagation algorithm (channel decoding), windowed encoding for low latency
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Source encoding via modified belief propagation algorithm (channel decoding), \textit{windowed encoding} for low latency

Source decoding via channel encoding
Coupling of Low-Density Generator Matrix Codes

(a) LDGM-BC

(b) SC-LDGM code: time $t$

(c) SC-LDGM code: time $t + 1$

$W = 3$

encoded bits
Results: Symmetric Bernoulli Source

![Graph showing deviation from RD limit vs latency for different values of L (32, 45, 55, 85, 100).]
Results: Symmetric Bernoulli Source

![Graph showing distortion vs latency for different SC(4, 8) configurations with varying M values.](image)
Take Aways

- **Leveraging network gains** in canonical multi terminal problems by nested SC-LDPC codes
  - relaying, broadcast, and cooperative diversity scenarios

- Low-complexity **lossy and lossless compression** with SC-LDGM codes
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- **Leveraging network gains** in canonical multi terminal problems by nested SC-LDPC codes
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- Example applications which can benefit from network compression gains:
  - Distributed compression for CRANs in 5G
  - Distributed compression of phasor measurement units in wide area measurement systems

- **Open:**
  - Communication problem: Design of nested codes for $M>2$
  - Compression problem: Design of nested codes and universal codes
Follow Up...

- Y.-C. Liang, S. Rini, J. Kliewer: On the design of LDPC codes for joint decoding over the multiple access channel, Submitted to ITW 2016.