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The Quantitative Identification of Asset Bubbles – A Survey

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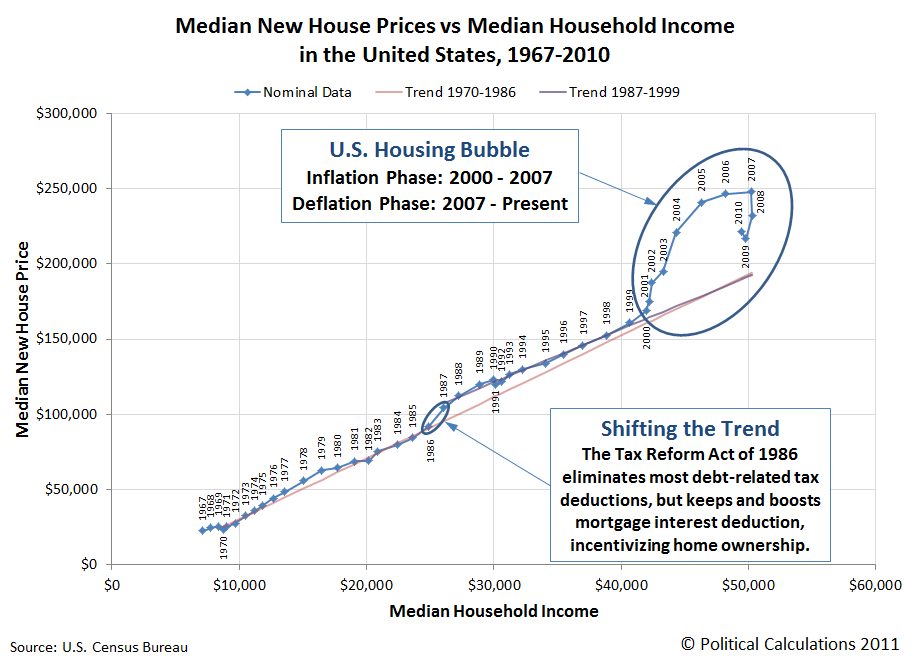
**1. Introduction**

http://knowledge.smu.edu.sg/images/archive/small.JPGThe damage wrecked by the bursting of asset bubbles can have a devastating impact on investors' fortunes and the welfare of a society. Krueger (2005) points out that the rise and fall of internet stock prices during the internet bubble destroyed about $8 trillion of shareholders’ wealth. More recently, the bursting of the housing bubble and toxic financial assets created worldwide financial crises, impacting many nations and their economies. Therefore, understanding the recent developments in the techniques of identifying asset bubbles is an important first step for researchers and policy makers to develop preemptive policy measures to ameliorate the negative impacts of speculative bubbles before they get too big and collapse.

An asset bubble occurs when the price of the asset is higher than the level that can be sustained by the economic fundamentals. However, as Alan Greenspan, the chairman of the U.S. Federal Reserve at that time, made his famous remark in December 1996, “How do we know when irrational exuberance has unduly escalated asset values?”[[1]](#footnote-1), the problem of identifying a bubble in the early stage is that it may be difficult to differentiate a boom from a bubble. In addition, as argued by Guenster *et al.* (2009) and Abreu and Brunnermeier (2003), when an asset bubble occurs, the price of the asset grows faster than fundamental values, in combination with a sudden acceleration of real price growth. Hence, it is also about how *fast* the asset price will increase than the fundamental values, which is related to the time series property of both asset price and the fundamentals over time. The question is – how fast is too fast?

From an economist perspective, “If you cannot quantify it, you don’t understand it.” This is also a similar saying in management that “If you cannot measure it, you cannot manage it.” In fact, if an asset bubble can be identified in the early stage of bubble formation, this can benefit different parties ranging from the academic and investment community to central bankers and policy makers. Therefore, to understand the recently developed methodologies to quantitatively identify the bubbles, or even the “irrational exuberance”, in terms of their time series properties is an important step toward the understanding and management of bubbles, which is the focus of this research.

There are many efforts to identify the asset bubbles. Some of them are less rigorous because they based on intuition and observations or graphical analysis of asset prices over time. For example, from the website of the Political calculation[[2]](#footnote-2), the author(s) compare medium new house prices and the median household income for the years from 1967 to 2010. The authors find that there is a shift of the trend after 1986, but, more importantly, they identify a U.S. housing bubble from 2000 to 2007 when the housing price deviates from the trajectory of the medium income over time, as shown in the following graph:

[](http://2.bp.blogspot.com/-Kgd1d8aWLrE/TufHJdD3jzI/AAAAAAAAExM/LiTiLgzDozs/s1600/median-new-house-prices-vs-median-household-income-us-1967-2010.png)

However, from a researcher’s perspective, such a graphical analysis of determining bubbles should only be viewed a convenient rule of thumb as it has potential problems. First, the medium house price can only serve as a proxy for the economic fundamentals underlying the housing price. This is because there are other factors that can influence the benchmark or long-run equilibrium housing prices, such as population growth (rate), house rent, stock market wealth, building or construction costs, mortgage rate, and the location of a city or metropolitan significant area (MSA), as discussed, for example, by Mikhed and Zemčík (2009), among others.

Second, the comparison between medium new house prices and the median household income is based on the yearly data, which is subject to data availability. In particular, the median household income is usually available only on the yearly basis, although it can be estimated between years. In this case, there is no good way to estimate the starting and ending time of a bubble if it occurs during the gaps of data. Hence, such an approach may have limited *ex ante* or real time pro-active policy implications in bubble formation process.

Third, and most importantly, such a technique is not rigorous in the sense that there is no quantitative characterization of the bubble. In particular, there is no test statistic such that statistics or econometrics can be applied to determine a statistically significant starting date and ending date of a bubble.[[3]](#footnote-3) The goal of this research is to survey the current researches to resolve such a problem of quantitatively identifying bubbles as well as some of the findings based on these methodologies.

Section 2 introduces some basic terminologies and concepts that are widely used in econometrics and statistics. Section 3 discusses the standard left-tailed Augmented Dickey-Fully (ADF) test as a way to determine the formation of a bubble. In addition, instead of a left-tailed ADF test, Section 4 introduces a recently developed right-tailed ADF test, the SADF test, to directly examine the evidence of asset bubbles. Section 5 discusses the development of alternative tests to SADF test and a generalized version of the PWY test, the GSADF test, which has higher power in detecting bubbles than other tests particularly when there are multiple bubbles in the same (long) period. In Section 6, we review three additional quantitative methods of determining asset bubbles. Section 7 presents the conclusion and discussions.

**2. Basis Terminologies and Concepts**[[4]](#footnote-4)

*2.1 Random Walk and Unit Root*

Suppose  is generated by the following model:

, where  is a stationary series with mean zero and variance. In this case, the first difference of, i.e., , is stationary with mean . This model is also known as the random-walk model with drift if  ≠ 0, or random walk without drift if  = 0.

*2.2 Unit Root Tests – ADF Test*

Empirically, whether  has a unit root or not needs to be tested. Suppose. A unit root is present if = 1. Then the regression can be re-written as follows: , where . This model can be tested for a unit root if = 0, which is called the Dickey Fuller Test. There are also two additional versions of the test,  and .[[5]](#footnote-5) In addition, a more general test is called the augmented Dickey-Fuller (ADF) test, which includes the possibility of lagged variables of. For example, the model can be modified as follows: , where. In the standard left-tailed ADF test, the null hypothesis is : = 0 (unit root behavior) against the alternative hypothesis : < 0 (stationary behavior).

*2.3 Co-integration*

A time series  is said to be integrated of order 1 or I(1) if is a stationary time series. A stationary time series is thus I(0). A random walk is a special case of an I(1) series or process. This is because if  is a random walk,  is a white noise, a special case of a stationary series. In addition, a times series  is integrated of order 2 or is I(2) if  is I(1). In addition, if  is I(1) and  is I(0), then their sum = +  is I(1).

Suppose that  is I(1) and  is also I(1), then  and  are co-integrated if there exists a  such that  –  is I(0), which can be denoted as CI(1, 1). In other words, the regression equation  =  +  is meaningful because  and  do not drift away from each other over time, which implies that there is a long-run equilibrium relationship between them.

In addition, if  and  are co-integrated such that  =  –  is I(0), then the unit root test can be applied to . The null hypothesis is :  has a unit root or  and  are not co-integrated, against the alternative hypothesis, :  and  are co-integrated.

For the purpose of identifying bubbles, the price levels in the long run equilibrium are usually assumed to be determined by the economic fundamentals. As a result, a bubble occurs only if the price level is not co-integrated with all the fundamentals, which implies that there is no long run equilibrium relationship between the price and fundamentals.

*2.4 Unit Root Test and Co-integration Test*

As discussed by Yiu and Jin (2012), “there are four possible scenarios of the unit root test on the asset price and fundamental value series: (A) asset price and fundamental values are both stationary in level; (B) asset price is stationary but fundamental value is non-stationary in level; (C) asset price is non-stationary and fundamental value is stationary in level; (D) asset price and fundamental value are both non-stationary in level. Scenario A indicates no bubble; Scenario B indicates an incorrect model; Scenario C suggests the presence of bubbles in the asset price; and Scenario D needs a co-integration test between asset price and its fundamental, assuming their first differences are stationary.”

**3. Unit Root Test – The ADF Test**

As the asset price can deviate from the fundamental values, Campbell and Shiller (1987) propose a method to use a unit root test as a first step to detect bubbles. If there is a bubble, the asset price and the fundamental value can be characterized by two possible cases. The first case is Scenario C discussed above in which the asset price is non-stationary (i.e., not mean reverting) in level, but the fundamental value is stationary. The second case is Scenario D, where both the asset price and fundamental value are non-stationary. Because of this, the second case calls for a con-integration test as second step. If a bubble is present, the asset price and its associated fundamental value will not be co-integrated (i.e., not having co-movement in the long run), assuming that they are both non-stationary in levels but stationary in first differences.

Similarly, based on the unit root and co-integration tests, Diba and Grossman (1988) point out that the bubble detection is equivalent to the identification of the explosive behavior in the gap between the asset price and its associated fundamentals. Since then, the standard unit root test (e.g. left-tailed ADF Test) and co-integration test on the price series and fundamental value series have been widely used for detecting asset bubbles because of this ease of implementation.

Nevertheless, when Diba and Grossman (1988) apply the unit root test to the real U.S. Standard and Poor’s Composite stock price index data from 1871 to 1986, they find that the stock prices are non-stationary in levels but stationary in differences. The authors also confirm the co-integration relationship between stock prices and dividends over the same period, which supports the argument that price did not diverge from the long-run fundamentals and thus giving evidence against the formation of the bubble in the stock market.

More recently, Ray and Ray (2009) explore the long term movement in the housing prices in selected American cities. By using the monthly S&P/Case-Shiller price index, they undertake the ADF test in each city. One important finding of their research is that there are three distinct cycles of convergence-divergence in the housing prices across the U.S. In particular, they find out that the U.S. housing prices broadly *converged* between January, 1987 and January, 2000. And then there is a rapid *divergence* in the prices, which continues until the middle of 2006. After about June of 2006, housing prices started to correct themselves, resulting in a *convergence* again. Therefore, housing price can diverge during the bubble, but will eventually converge again after the bubble burst.[[6]](#footnote-6)

Mikhed and Zemčík (2009) use aggregate data and panel data to determine whether recently high and subsequent rapid decreasing U.S. house prices reflect the economic fundamentals, such as personal income, population, house rent, stock market wealth, building costs, and mortgage rate. They believe that “a bubble in the price of housing may be identified if this price has a unit root, but housing demand and supply shifts are stationary or these shifts are not co-integrated with the price.” Hence, they also conclude that the “house price does not align with the fundamentals in sub-samples prior to 1996 and from 1997 to 2006, and “three peaks followed by a rapid decline can be identified in the house price series: 1979, 1989, and 2006, and “the most recent correction (a collapsed bubble) occurred around 2006.”

In addition, Clark and Coggin (2011) examine the existence of a U.S. housing price bubble using quarterly data from the first quarter of 1975 through the second quarter of 2005. The authors find that U.S. house prices and the fundamental economic variables are non-stationary unit root variables that are *not* co-integrated, even after allowing for structural breaks. These results confirm the existence of a U.S. housing price bubble for this period.

**4. The PWY Test**

*4.1 The Limitations of the Unit Root Test*

Despite the popularity of the ADF test, the results from the unit root test needs to be interpreted with caution. As discussed by Yiu and Jin (2012), “when the asset price and its fundamental value are found to be co-integrated, the null hypothesis of no bubble is confirmed. However, the reverse may not be necessarily true, because the presence of a rational bubble may be one of the many possible reasons. Other possible factors include, for example, the non-stationary nature of unobservable variables.”

Moreover, because the standard ADF test “utilizes a linear model to detect any non-linear growth of the bubble component, the power of detecting explosive bubble behavior and identifying the origination and collapse of a bubble, particularly the collapse” is limited. Similarly, as argued by Evan (1991), as well as Phillips, Wu, and Yu (2011a), “standard unit root test and con-integration tests are inappropriate tools for detecting bubble behavior because they cannot effectively distinguish between a stationary process and a periodically collapsing bubble model.” This is because patterns of periodically collapsing bubbles in the data look more like data generated from an I(1) process, such as a unit root, or even a stationary auto-regression process than a potentially explosive process.[[7]](#footnote-7) In addition, even if the process is explosive, within a limited time period when a bubble is first developing, the magnitude of the bubble may be small relative to the fundamental(s), which may result in underestimating the true growth rate of the bubble.

*4.2 The PWY Test-SADF Test*

To overcome these problems, Phillips, Wu, and Yu (2011)[[8]](#footnote-8) propose a newly developed method, the PWY test, to determine the existence of an asset bubble. Instead of the standard left-tailed ADF test, PWY method arranges the right-tailed ADF test to both the price and dividend. By using the same symbols as in Section 2 such that, the null hypothesis is : *δ* = 0 (unit root behavior) against the alternative hypothesis : *δ*  > 0 (*mildly* explosive). Therefore, the PWY test looks directly for evidence of non-linear explosive behavior of the asset price.

Also, in terms of forward recursive regressions, the sup ADF test (SADF test) is repeated using subsets of the sample data incremented by one observation at each time. The origination of a bubble is dated as the first recursion for which the value of the *t*-statistic of the estimated *ρ* or  equal to or above the right side critical value of a particular significance level. The collapse date is identified as the first subsequent recursion for which the *t*-statistic decreases to the level that is equal to or below the critical value. In other words, the SADF test not only can determine the origination of a bubble by estimating the date of a regime switch from a random walk to an explosive process, but also can determine the collapse of a bubble by estimating the date when the there is a regime switch from an explosive process to a random walk. Therefore, the forward recursive procedure is stronger than the standard ADF test or con-integration test in identifying the beginning and collapse of an asset bubble.

In addition, as the authors seek to quantify the “exuberance”, coined by Greenspan (1996), in terms of a mildly explosive auto-regressive behavior, they examine the internet bubble for the Nasdaq price index, adjusted by inflation. By using 5% significance level as the critical value, their results show that the Nasdaq price level is explosive, while the dividend level is not, which supports the existence of a bubble. They also determine that the internet bubble emerged from July 1995 (until February 2001, and then started again from April 2001) and collapsed in July 2001, where the prices had by then already collapsed.

*4.3 More Applications of the PWY Test*

*4.3.1 Recent U.S. Bubbles*

In a recent study, Caballero *et al*. (2008) argue that “the internet bubble in the 1990s, the asset bubbles over 2005 to 2006, the subprime crisis in 2007, and the commodity bubbles in 2008 are closely related.” To econometrically test such a statement, Phillips and Yu (2011) apply the PWY test with some modification and improvement to determine the origination time and ending time of three financial time series during the subprime crisis, including “a financial asset price (the house price index), a commodity price (the crude oil price), and one bond price (the spread between Baa and Aaa).” The authors not only identify different bubble behavior statistically, but also provide consistent dating of the origination and collapse of the bubbles.

The empirical estimates of the origination and collapse of these financial variables show that there is some migration or transmission mechanism among them. In particular, as discussed above, Phillips, Wu, and Yu (2011) estimate that the origination date of the internet bubble was July 1995, and the termination date was July 2001. Then a real estate bubble emerged in February 2002 and collapsed in December 2007, soon after the subprime crisis erupted. Then the phenomenon migrated selectively to both the bond market and the commodity market. The bubble in the crude oil market started in March 2008 and ended in July 2008. Then the bubble in bond market appeared on September 22, 2008, and collapsed in April 20, 2009.

Their “estimates suggest that the bubbles emerged in the housing market before the subprime crisis, and collapsed with the subprime crisis. The bubble then migrated from the housing market to the selective commodity market and the bond market after the crisis erupted into the public arena. All these bubbles collapsed as the financial crisis impacted the real economic activity.” Therefore, “the estimated sequence is broadly consistent with the statement made by Caballero *et al*. (2008).”

Most importantly, the PWY test “can be used to provide early warning diagnostics for market exuberance as they provide consistent tests for mildly explosive behavior. Such diagnostics may assist policy makers in framing early preventive monetary policy responses or other regulatory actions or interventions” to alleviate the possible negative impacts of the speculative asset bubbles.

*4.3.2 Hong Kong Housing Bubbles*

Yi and Jin (2012) apply the PWY Test to identify housing bubbles in the Hong Kong residential property market. Unlike the Phillips, Wu, and Yu (2011a) who test the price and fundamental series separately, Yi and Jin (2012) examine the differential between the property price and the fundamental, i.e., the rent, directly based on the price/rent ratio, which is calculated as the log difference between the real property price index and real rent index of the Hong Kong residential property market. The log difference is expected to have an explosive feature if an asset bubble occurs. One reason for doing so is that both the price and rent are non-stationary.

Accordingly, “the origination of the bubble is identified in February 1997 when the *t*-statistic for the first time exceeds both the 5% and 10% critical values. However, the collapse date of the bubble depends on which significance level is used. If the 5% significance level of the *t*-statistic is used, the end date was July 1997 when the real price dropped 4.1% from the peak in May 1997. On the other hand, if the 10% significance level is used, the end date was November 1997 when the real price fell 9.2% from the peak.”

In addition, the method detects explosive growth of the price-rent differential in Hong Kong since July 2009, indicating strong upward price pressure. “During the first quarter of 2011, the result also shows clear explosive behavior of the differential which signals an asset-bubble formation. Most importantly, their results show the potential of the PWY Test to be used of timely monitoring of bubble formation in asset markets since the method will provide updated indications of bubble formation, once new data become available, i.e., with time lag of about six weeks.” Although the identification of the starting date of a bubble cannot lead to the *prediction* of the collapse time of the bubble, it can give policy makers some early warning so that pro-active policy actions can be taken in a timely manner.

**5. PWY Test and Other Associated Tests**

*5.1 The PWY Test vs. Alternative Tests*

By using the PWY test as the benchmark, Homm and Breitung (2012) compare several alternative modified tests, originally proposed by Bhargava (1986), Busetti and Taylor (2004), and Kim (2000), for rational bubbles and investigate the power properties of these tests. The focus is on the case where bubble detection is reduced to testing for a regime change from a random walk to an explosive process. In addition, the authors apply the modified sequential Chow-Test for structural breaks. When there is only one regime change, the sequential Chow-Test and the modified Busetti and Taylor (2005) test procedures exhibit the highest power when the change from I(1) to explosive occurs late in the sample.

Moreover, with regard to the estimation of the date when a bubble emerges, a breakpoint estimator derived from the sequential Chow-Test turns out to be the most accurate one when compared to the PWY test. However, this result does not hold if there are multiple bubble and crashes, i.e., multiple structural breaks. In that case, the PWY test is much more robust against multiple structural breaks than all other tests.

The authors also analyze the Nasdaq composite index and various other financial time series, such as S&P 500, Nikkei 225, Hang Seng, and Shanhai. As a result, the PWY test provides strong evidence for the bubble presence in the Nasdaq index and other stock markets. Similar results are also derived by using the sequential Chow-Test and Busetti and Taylor (2004) test, thus reinforcing the results of the PWY test.

Furthermore, the authors also redesigned all tests as monitoring procedures, with the focus on FLUC (similar to the PWY test) and CUSUM procedures as a real time monitoring approach to detect the origination date of the emerging bubbles. The estimation of the origination date for the Nasdaq bubble to emerge is June 1995. Based on the monthly data, the authors also estimate the origination dates of bubbles for different stock markets. They are 1990-10-31 for S&P 500, 1982-10-03 for Nikkei 225, 2003-03-31 for Hang Seng, and 2005-12-02 for Shanghai. Likewise, the estimated origination dates of housing bubbles are 1999-06-15 for the U.S., 1997Q4 for Spain, 1999-01-15 for the U.K., and 1985 for Japan.

*5.2 The Test for Multiple Bubbles-The GSADF Test*

Gilbert (2010) applies the PWY text to commodity futures prices and discusses the method’s power of detecting multiple bubbles. The author finds that PWY test is not a very effective test for multiple bubbles, in particular if the explosive behavior is more pronounced in the first bubble. In addition, Phillips, Shi, and Yu (2012) admit that the “complexity of the non-linear structure inherent in multiple bubble phenomena within the same sample period makes econometric analysis particularly difficult.” They also “show how the testing procedure and dating algorithm of PWY test are affected by multiple bubbles and may fail to be consistent.” Therefore, the authors generalize the SADF test to GSADF test to overcome such difficulty. Their simulation results show that the GSADF test significantly improves discriminatory power in detecting multiple bubbles and collapses.

Phillips, Shi, and Yu (2012) also conduct empirical applications of the GSADF test to price-dividend ratio of S&P 500 stock market monthly data from January 1871 to December 2010. The results identify many key historical episodes of *both* exuberance and collapse over this period, including the great crash episode (1929M01-M09), the postwar boom in 1954 (1954M12-1955M12), black Monday in October 1987 (1987M02-M09), the dot.com bubble (1995M12-2001M06), and the subprime mortgage crisis (2008M10-M09),” for the episodes with the durations greater than or equal to half of year. For the other episodes that are less than six months, the authors also identify the “explosive recovery phase from the panic of 1873 (1879M10-1880M02), the banking panic of 1907 (1907M10-M11), and the 1974 stock market crash (1974M09).” In contrast, the SADF test (i.e., the PWY test) only identifies two explosive periods – the recovery phase of the panic of 1873 (1879M10-1880M04) and the internet bubble (1997M07-2001M08).

The authors also apply the CUSUM procedure proposed by Homm and Beritung (2012) to the same data to detect the emergence of a bubble. The results show that CUSUM procedure identifies some bubble episodes for periods before 1990. However, “for the past-1990 sample, the procedure detects only the great crash and the dot.com bubble episodes.” Hence, CUSUM does not provide early warning alert or acknowledgement of black Monday in October 1987 or the subprime mortgage crisis in 2008. As the new recursive procedures GSADF is more sensitive in identifying multiple bubbles and collapses than the SADF or the CUSUM, GSADF therefore is a better tool for the central bankers and fiscal regulators for practical implications and surveillance strategies.

**6. Other Tests for Detecting Bubbles**

*6.1 State-space Model with Markov-Switching*

Similar to the PWY test, there are other studies that implement direct tests for speculative bubbles by explicitly formulating the existence of a bubble in the alternative hypothesis. Examples of such direct test procedures can be found in the research by, for example, West (1987) and Wu (1997), among others. By following such an approach of research, Al-Anaswah and Wilfling (2011) adopt a state-space model with Markov-switching methodology to detect speculative price bubbles in the stock market, originally used by researchers, such as Kim and Nelson (1999), among others, in the literature of detecting the turning points of business cycles. Hence, Al-Anaswah and Wilfling (2011) utilize a two-regime Markov-switching specification for the unobservable bubble process. One regime is when the bubble survives, while the other one is when the bubble collapses. A bursting bubble is identified if the two regimes can be separated.

The authors test artificial data and the real world bubbles, specifically, the famous bubbles discussed by Kindleberger and Aliber (2005), such as the Dutch Tulip Bulb Bubble in 1636, the South See Bubble in 1720, the Mississippi Bubble in 1720, and so on. The authors also test some other famous bubbles, and are able to identify most, but not all, bubbles, such as the Black Monday, and the burst of the internet bubble. In addition, the authors use the entire monthly data set of U.S. stock market from January 1871 to June 2004, but cannot find significant results until they separate the data into three sub-periods. Therefore, despite its ability to detect bubbles, this technique may not have the same power in detecting and particularly date-stamping bubbles as the GSADF procedure.

*6.2 The Duration Dependence Test*

As discussed by Mokhtar *et al.* (2006), “the majority of published studies examining the existence of speculative bubbles are concentrated on the techniques used to detect rational speculative bubbles in the stock market.” “These techniques can be grouped into four main categories: (1) tests for bubble premiums, (2) tests for excess volatility, and (3) tests for the con-integration of dividends and prices, and (4) the duration dependence test.”

For the research in category (1), “a bubble premium is the excess returns the investor demand above the fundamental return in the presence of speculative bubble. This return has an explosive nature as it increases geometrically through time, and incorporates the actual excess return of the stock over the risk free rate.” However, “the broad consensus of this literature is that tests for the presence of a bubble premium face serious problems and may not be able to prove or adequately disprove the existence of rational speculative bubble.” [[9]](#footnote-9)

For category (2), if a speculative bubble is present, the variance of the stock price will be higher than the variance of the fundamental price. Examples can be found in the studies by Shiller (1981), West (1987), and Wu (1997), among many others. For category (3), it is obvious that Mokhtar *et al.* (2006) were not aware of the recently developed PWY test (in 2009) and other tests that will solve the underlying problems of the unit-root test and the co-integration test that detect bubbles indirectly. Therefore, the authors focus on category (4).

From a behavioral finance perspective, the authors “investigate the presence of rational speculative bubbles in Malaysian stock market by employing the duration dependence test based on the Weibull’s hazard model and Log Logistic hazard model.” From the abnormal monthly real returns and three time frames – before (1994-1996), during (1997-1998), and after (1999-2003) the Asian financial crisis in 1997, the authors confirm the existence of rational speculative bubbles in Malaysian stock market before and after the crisis, although the post-crisis bubble is smaller than the pre-crisis bubble. This weakness of this approach, however, is its inability to date-stamp the origination and collapse of the bubble, which also can be resolved with the utilization of the recently developed SADF or GSADF test.

*6.3* *Asset Price Volatility*

Recently, in the field of mathematical finance, Jarrow *et al.* (2011) propose a new methodology to detect bubbles based on the asset price volatility. They characterize the asset price bubbles as frictionless, competitive, and continuous trading models using the arbitrage-free martingale pricing technology. The asset price process is defined in terms of a standard stochastic differential equation, which is driven by the Brownian motion. The determination of a bubble depends on whether the price process under a risk neutral measure is a martingale or a strict local martingale. The difference between them hinges on the asymptotic behavior of the asset’s price volatility. If the volatility of the asset price is high, then a bubble exists.

The authors also apply the same methodology to several stock prices during the internet bubble, namely, Lastminute.com, eToys, Infospace, and Geocities. For the case of eToys, the result is inconclusive about whether a bubble has occurred. For Lastminute.com and Infospace, the methodology supports the presence of a price bubble. However, for Geocities, the result does not show the existence of a price bubble. Therefore, this research should be viewed as an early effort to detect bubbles in real time. More applications need to be done in terms of testing the famous historical bubbles instead of the stock prices of a few firms.

**7. Conclusion and Discussions**

*7.1 The Tests and the Identification of Bubbles*

In this research, different methodologies to quantitatively detecting the bubbles are reviewed, including the ADF test, SADF (PWY) Test, GSADF Test, the sequential Chow-Test, Bhargava (1986) test, Busetti and Taylor (2004) test, and Kim (2000) test, the CUSUM test, the Test based on state-space model with Markov Switching, duration dependence test, and the asset price volatility test from mathematical finance. The applications of these tests have successfully indentified most, if not all, the famous stock price bubbles in history, such as Nasdaq (1995), S&P 500 (1999), Nikkei 225 (1982), Hang Seng (2003), and Shanghai (2005), as well as those in Malaysia, Indonesian, and other counties. Similarly, the housing bubbles in different nations are identified, including the U.S. (1999), Spain (1997Q4), the U.K. (1999), and Japan (1985).

In addition, based on the GSADF Test, both the origination and collapse dates are identified in many famous episodes in history, including the great crash episode (1929M01-M09), the postwar boom in 1954 (1954M12-1955M12), black Monday in October 1987 (1987M02-M09), the dot.com internet bubble (1995M12-2001M06), and the subprime mortgage crisis (2008M10-M09), the recovery phase from the panic of 1873 (1879M10-1880M02), the banking panic of 1907 (1907M10-M11), and the 1974 stock market crash (1974M09).” Sometimes, different tests may give slightly different results. For example, the SADF test identifies two explosive periods – the recovery phase of the panic of 1873 (1879M10-1880M04) and the internet bubble (1997M07-2001M08).

*7.2 More Comparisons and Policy Options*

In the old days when there are no statistically reliable methods to detect the bubbles or “exuberance”, either rational or irrational, people can only *intuitively* or *graphically* “detect” a bubble when the asset price continues to increase unreasonably in comparison with the economic fundamentals. Although one may argue that “When you are in a bubble, you know it.”, such a statement is not very useful for policy makers. By the time the asset price reaches its historical peak, an imminent bubble implosion and its destabilizing effect to the economy will be inevitable. At that time, there is very little that policymakers can do to smoothly deflate the bubble. This is why the Fed used to focus on cleaning the mess after the collapse of a bubble, but has started to change its policy towards early pro-active policy actions to smooth out the bubbles without causing damages to the real sectors of the economy.

Today, with these newly developed econometric and mathematical techniques to monitor asset prices, it is no longer impossible to rigorously detect a bubble before it collapses. However, just because a bubble has been identified, it does not mean that the bubble is also due to pop any time soon. Therefore, the next step of researchers and policymakers is (1) to further compare these tests in terms of their strengths and weaknesses such that we can find the most efficient ways to combine these tests, given the data availability, to have (a) reliable early warnings for both the origination and *predicted* collapsing dates of the bubbles, (b) the mechanisms to monitor bubbles in real time, and (2) to develop policy options and execute these policy measures in a timely manner for maximum effect based on the results of these the statistical and mathematical techniques.

*7.3 Unit Root Test vs. PWY Test vs. Other Tests*

As the asset price may change from the random walk to explosive behavior when a bubble emerges or from explosive behavior to random walk after the collapse of a bubble, the tests reviewed in this research are not necessarily contradictory to each other. In fact, these tests should be viewed as *complements* instead of *substitutes*. It is true that the presence of the random walk or unit root may not be the clear evidence of an emerging bubble. However, it may serve as an early warning if the asset price continues to become divergent or explosive statistically later on based on the SADF test or other related tests. Similarly, if an explosive asset price starts to show patterns that are similar to the unit root, once again, it may serve as an early warning that the bubble is going to collapse soon, which can be verified later on with rigorous test, such as SADF.

Similarly, the GSADF test is more sensitive than CUSUM and SADF in detecting asset bubbles, particularly when there are multiple bubbles. However, if the policy makers or the regulators are less risk averse towards a bubble, they may lean towards the prediction of later bubble formation from, for example, CUSUM and SADF instead of GSADF. Therefore, the policy makers, such as the Fed, and the regulator, such as the federal government, need to have a team of experts that can statistically determine and predict the asset bubbles, as well as the consensus on the criterion of when the action should be taken. Although every bubble is different, bubbles and crashes can be correlated with each other. Therefore, it is important that both the Fed and the federal government need to have a constant monitoring system of bubbles, and well-prepared strategies in every stage of every bubble to deal with the different predictions from these tests in terms of fiscal and monetary policies or regulatory measures.

*7.4 Significance Levels*

Moreover, as different confidence or significance levels are not necessarily contradictory to each other, the choice of the significance level may also depend on the risk aversion of the policy maker toward the bubble. While a 1% significance level may be the best predictor of a bubble, a 10% significance level may provide the earliest warning of a bubble, although it could be wrong. The middle way of course is the 5% level of significance. Once again, the Fed and the federal government should have well-prepared policy options at every significance level.

*7.5 Efficiency of the Markets*

As discussed by Mokhtar *et al.* (2006), one of the reasons that the Malaysian bubble in the post-Asian crisis period is smaller than the one in the pre-crisis period is that the market has become more efficient. Particularly, after the Asian crisis, a few actions were taken in Malaysia, including “ownership rules, liberalization of investment rules, improvement of banking sector prudential regulation, capital market reform, corporate governance reforms and corporate restructuring. These measures successfully brought confidence among local as well as foreign investors towards the Malaysian stock market.” Hence, the changes of policies to build up the economic fundamentals through capital and currency control have improved market efficiency, and thus reduce the size of the post-crisis bubble.

In the case of the U.S., domestically, one current policy focus of the regulator is Dodd-Frank Act which seeks to strengthen the regulation against financial institutions and prevent the next financial bubble. The other policy focus is to improve the current economic situations without increasing the soaring national debt. Internationally, the euro crisis is one prime example why bubbles and crashes across countries can be correlated. How to improve the economic efficiency of a country or countries and prevent or alleviate the next bubble will continue to be one important topic for researchers and policy makers in the future.

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1. See Greenspan (1996) for details. [↑](#footnote-ref-1)
2. See <http://politicalcalculations.blogspot.com/2010/02/better-method-for-detecting-housing.html> and <http://politicalcalculations.blogspot.com/2011/12/revisiting-us-housing-bubble.html> for details. [↑](#footnote-ref-2)
3. There are several ways to statistically test the relationship between the housing price and medium income, which will be discussed in Sections 3 & 4. [↑](#footnote-ref-3)
4. Sections 2.1-2.3 are partially based on Maddala (1992). [↑](#footnote-ref-4)
5. See Dickey and Fuller (1979) for details. [↑](#footnote-ref-5)
6. This result is consistent with the theoretical model proposed by Chou (2012) in which the housing price will converge with oscillation before the bubble, diverge with oscillation during the bubble, and then diverge without oscillation after the price starts to decrease. [↑](#footnote-ref-6)
7. See <http://knowledge.smu.edu.sg/article.cfm?articleid=1385> and Mokhtar *et al.* (2006) for similar discussions. [↑](#footnote-ref-7)
8. The same authors first proposed the PWY test in a Yale University Working paper in 2009. [↑](#footnote-ref-8)
9. See the discussion of Liu *et al.* (1995) for further interest. [↑](#footnote-ref-9)